Unified Engineering Problem Set **Lectures: M18, M19, M20, M21**
Week 13 Spring, 2008 **Units: M5.1, M5.2, M5.3, M5.4**

M13.1 ***(10 points)*** A spring is basically a device for storing energy. Therefore, one key consideration in the design of a spring is the maximum energy that can be stored. A second consideration is that the spring should return to its original configuration when unloaded. Thus, the material cannot yield. Consider a cantilevered beam loaded by a downward tip load (P) to be a rotational spring. The beam is of fixed length (L) and fixed width (w), while its thickness (t) can vary.

(a) For a given thickness, express the total energy stored in the beam as a function of the applied tip load (P), the modulus of the material, and the three geometric parameters (L, w, t).

(b) Identify the combination of material properties that maximize energy storage capacity (without yielding) for:
  (i) a given thickness of material;
  (ii) a given mass of material;
  (iii) a given cost of material.

(c) Choose amongst the following materials for each of the three criteria listed in part (b). Comment as appropriate.

Material	Modulus E [GPa}	Yield Stress σ_y [MPa]	Density ρ [Mg/m^3]	Price c [$/kg]
Al alloy	70.0	500	2.7	2.5
Spring Steel	210	2400	8.0	3.5
Wood	10.0	70.0	0.5	1.0
Titanium	116	1400	4.5	13
Glass/Epoxy	140	400	2.0	12
Graphite/Epoxy	100	650	1.5	200

M13.2 ***(10 points)*** A specially-assembled design team is considering the design of a joint intersection of a particular structure. They have identified some key locations in the joint where the stresses are greatest and expressed these as proportional to some loading characteristic, q. This loading characteristic is related to the overall structural loading parameters, but is not expressed directly in terms of such due to *certain concerns*. The four stress conditions are:

<u>Condition A</u>:

$\sigma_{11} = 4q$ $\quad\sigma_{12} = 0$
$\sigma_{22} = -2q$ $\quad\sigma_{13} = 0$
$\sigma_{33} = q$ $\quad\sigma_{23} = 0$

<u>Condition B</u>:

$\sigma_{11} = q$ $\quad\sigma_{12} = 2q$
$\sigma_{22} = 4q$ $\quad\sigma_{13} = 0$
$\sigma_{33} = 0.5q$ $\quad\sigma_{23} = 0$

<u>Condition C</u>:

$\sigma_{11} = -3q$ $\quad\sigma_{12} = 0$
$\sigma_{22} = -3q$ $\quad\sigma_{13} = 0$
$\sigma_{33} = -3q$ $\quad\sigma_{23} = 0$

<u>Condition D</u>:

$\sigma_{11} = q$ $\quad\sigma_{12} = 0$
$\sigma_{22} = 2q$ $\quad\sigma_{13} = 0$
$\sigma_{33} = 4q$ $\quad\sigma_{23} = 0$

The designers are currently considering an aluminum alloy with a yield stress of 300 MPa.

(a) Using Tresca's yield criterion, calculate the value of the loading characteristic, q, for the onset of yielding and the associated plane on which yield would occur for each.

(b) Repeat this calculation using the von Mises criterion.

(c) Comment on the overall results.

M13.3 ***(10 points)*** Areas of the lower wing skin of an airplane wing can be modeled to first order as a plate of material with superposed longitudinal (along the wing) and transverse loads due to climb (>+1), level flight (+1), and descend (< +1) flight with additional torsional loads due to maneuvers. Consider a mid-sized airplane with a mid-wing panel with a thickness of 0.125 inches. Given the "skeleton" box construction of the wing that includes the spars, the chords, and the stiffeners, the following stresses are those actually carried by the skin. The stresses due to level flight are:

$$\sigma_{longitudinal} = 15 \text{ ksi}$$
$$\sigma_{transverse} = 6 \text{ ksi}$$

These conditions for level flight come from n=1 operation producing the stresses noted. Stresses for climb and descend are determined based on the ratio of n for those operations with the n=1 for level flight. Shear stresses are produced via the maneuvering operations and are related to a parameter, m, with the value of shear stress for m=1 equal to 10 ksi. Only consider operational conditions with values of n and m greater than or equal to 0.

The wing is made of 2024 aluminum that has a modulus of 10.1 Msi, a Poisson's ratio of 0.3, and a yield stress of 50 ksi.

(a) Consider a piece of material on the wing that is stressed as noted. Using limit condition defined by the point of yield in the structure and the Tresca failure criterion, determine an "operating envelope" for this piece of material. Use axes of the two loading parameters, n and m.

(b) A *damage tolerant* approach is now taken such that the material must tolerate a through-crack of 0.40 inches in length that can be detected nondestructively in scheduled inspection intervals. The fracture toughness of the 2024 aluminum is 31 ksi $in^{1/2}$. In applying the fracture mechanics criterion, ignore all stresses except that perpendicular to the crack. Determine the "operating envelope" for this same piece of material for limit condition defined by the damage tolerant approach such that no crack propagates up to and at limit condition.

(c) Compare the two sets of results and make relevant comments.

M13.4 This problem is provided as a STUDY HELPER in relation to Units M5.1 and M5.2 (mainly the latter).

It is *Not* to be handed in.

Eight questions and eight answers are provided. You are to match the answers to the appropriate questions.

Reading the assigned sections and chapters for these units will be of help.

Questions:

1. In ancient times, swordsmiths would use bronze, an alloy of copper and tin, to make their swords. Why did they use this alloy rather than pure copper?

2. These swordsmiths would manufacture these swords by repeated hammering rather than by melting the metal and then casting the molten metal to the appropriate shape. What did this repeated hammering achieve?

3. Why can the yield strength of very fine (approximately 1 μm in diameter) needlelike crystals approach the theoretical strength (modulus/15), whereas bulk specimens of the metal do not come close?

4. Aluminum alloys are generally not considered viable for use on critically exposed surfaces of supersonic alloys. Why not?

5. Thin metals wires (such as electrical connectors and cables) are made by *drawing*, i.e. pulling, a thicker cross-section through a die. Why is there an upper limit to the reduction in the cross-sectional area (known as the draw ratio) that can be achieved in this process?

6. Why do wires become hot during the drawing process?

7. A metal developer is trying to strengthen an aluminum alloy by adding aluminum oxide powder. Why do the company researchers find that a smaller particle size produces a higher yield strength for a given volume fraction of powder?

8. Why do pure metals generally have lower yield stresses than ceramics?

Answers:

A. Several factors contribute to this. One, there are less likely to be imperfections, therefore less dislocations are available to cause yield. Two, overall directions may be oriented favorably relative to the loading direction such that the shear stresses acting on the slip planes will be small. Three, the surface provides a barrier to dislocation motion.

B. The cross-section is reduced by plastic deformation and this deformation is achieved by applying a uniaxial tensile stress in the wires. A plastic instability can result, thereby causing the material to neck down and rupture.

C. Yield is determined by dislocation motion. Metals generally have more close-packed crystal structure than ceramics. Therefore, there are more slip planes on which dislocations can glide in metals. Furthermore, ceramics are often covalently bonded giving a large intrinsic lattice resistance to dislocations since covalent bonds are more directional than metallic bonds and thus cannot switch as easily between neighboring atoms.

D. The atoms of the second material locally distort the FCC crystal lattice of the primary material. This has the effect of "roughening" the slip planes which increases the resistance to dislocation glide. This results in an increase in the yield stress thereby producing a harder material.

E. Particles "pin" dislocations, thereby increasing the resistance to glide along the slip planes. The increase in the shear yield stress is inversely proportional to the spacing of the particles. A smaller size for a given volume fraction results in more particles with closer spacing.

F. Nearly all of the mechanical work done to plastically deform the material is converted to heat as plastic deformation is an irreversible process.

G. This is a form of work hardening. Extra hardness can be obtained by work hardening by introducing plastic deformation. More dislocations are introduced. These interfere with each other and further increase the yield stress.

H. The material is exposed to sustained temperature in the vicinity of 300°F and higher. Creep can occur at these temperature under sustained load.